Homeward Bound

A GPS receiver told you that your home was 15.0 km at a direction of 40° north of west, but the only path led directly north. If you took that path and walked 10 km, how far and in what direction would you then have to walk in a straight line to reach your home?

➥ *Look at the Example Problem on page 75 for the answer.*

CHAPTER

4 Vector Addition

Finally, after hours of hiking and clambering up rocks, you've reached your destination. The scene you have been anticipating unfolds before you. It's the reward for the long trek that has brought you here, and it's yours to enjoy.

But no matter how inviting the scene, eventually the time comes when you need to think about the journey home. It's very easy to lose track of directions in a region so vast. Suddenly the landscape looks the same in every direction. Exactly where are you, and in which direction is the way home?

Unlike earlier adventurers who relied on the position of the sun and stars, you rely on a GPS receiver to help you find your way home. The small, handheld device can pinpoint your location with an accuracy of 50 meters. The GPS receiver uses signals from two dozen satellites of the Global Positioning System (GPS) to determine location. The satellites are located in regular, stationary orbits around the world. Each has a different displacement from the receiver. Thus, synchronized pulses transmitted from the satellites reach a single receiver at different times. The GPS receiver translates the time differentials into data that provide the position of the receiver. From that position, you can determine the displacement—how far, and in what direction—you need to travel to get home.

Recall from Chapter 3 that displacement is a vector quantity. Like all vectors, displacement has both magnitude (distance) and direction. In this chapter, you'll learn how to represent vectors and how to combine them in order to solve problems such as finding your way home. In preparation for this chapter, you may want to look again at Appendix A and review some mathematical tools, such as the Pythagorean theorem and trigonometric ratios.

WHAT YOU'LL LEARN

- You will represent vector quantities graphically and algebraically.
- You will determine the sum of vectors both graphically and algebraically.

WHY IT'S IMPORTANT

- Airplane pilots would find it difficult or impossible to locate their intended airport or estimate their time of arrival without taking into account the vectors that describe both the plane's velocity with respect to the air and the velocity of the air (winds) with respect to the ground.

PHYSICS Online

To find out more about vectors, visit the Glencoe Science Web site at **science.glencoe.com**

CLICK HERE

4.1 Properties of Vectors

OBJECTIVES

- **Determine** graphically the sum of two or more vectors.
- **Solve** problems of relative velocity.

You've learned that vectors have both a size, or magnitude, and a direction. For some vector quantities, the magnitude is so useful that it has been given its own name. For example, the magnitude of velocity is speed, and the magnitude of displacement is distance. The magnitude of a vector is always a positive quantity; a car can't have a negative speed, that is, a speed less than zero. But, vectors can have both positive and negative directions. In order to specify the direction of a vector, it's necessary to define a coordinate system. For now, the direction of vectors will be defined by the familiar set of directions associated with a compass: north, south, east, and west and the intermediate compass points such as northeast or southwest.

Representing Vector Quantities

In Chapter 3, you learned that vector quantities can be represented by an arrow, or an arrow-tipped line segment. Such an arrow, having a specified length and direction, is called a **graphical representation** of a vector. You will use this representation when drawing vector diagrams. The arrow is drawn to scale so that its length represents the magnitude of the vector, and the arrow points in the specified direction of the vector.

In printed materials, an **algebraic representation** of a vector is often used. This representation is an italicized letter in boldface type. For example, a displacement can be represented by the expression ***d*** = 50 km, southwest. *d* = 50 km designates only the magnitude of the vector.

The resultant vector Two displacements are equal when the two distances and directions are the same. For example, the two displacement vectors, ***A*** and ***B*,** as shown in **Figure 4–1,** are equal. Even though they don't begin or end at the same point, they have the same length and direction. This property of vectors makes it possible to move vectors graphically for the purpose of adding or subtracting them. **Figure 4–1** also shows two unequal vectors, ***C*** and ***D*.** Although they happen to start at the same position, they have different directions.

Color Conventions

- Displacement vectors are **green.**
- Velocity vectors are **red.**

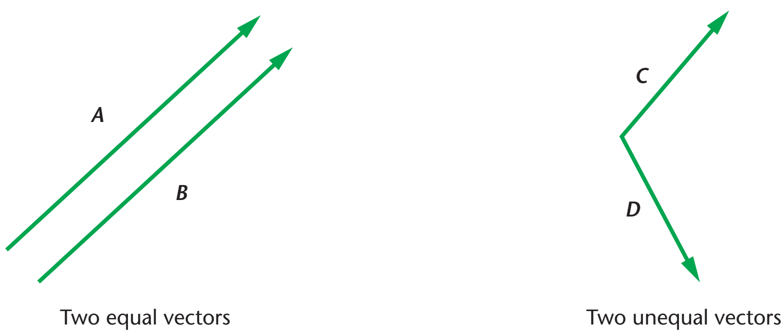


FIGURE 4–1 Although they do not start at the same point, ***A*** and ***B*** are equal because they have the same length and direction.

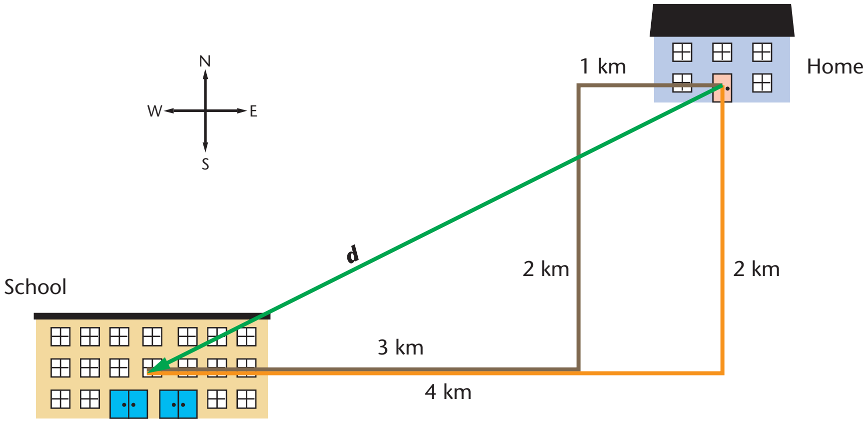


FIGURE 4–2 Your displacement from home to school is the same regardless of which route you take.

Recall that a displacement is a change in position. No matter what route you take from home to school, your displacement is the same. **Figure 4–2** shows some paths you could take. You could first walk 2 km south and then 4 km west and arrive at school, or you could travel 1 km west, then 2 km south, and then 3 km west. In each case, the displacement vector, ***d,*** shown in **Figure 4–2,** is the same. This displacement vector is called a **resultant vector.** A resultant is a vector that is equal to the sum of two or more vectors. In this section, you will learn two methods of adding vectors to find the resultant vector.

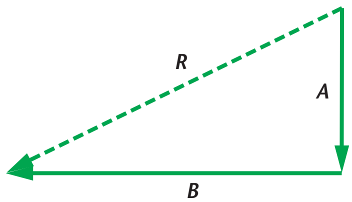


FIGURE 4–3 The length of ***R*** is proportional to the actual straight-line distance from home to school, and its direction is the direction of the displacement.

Graphical Addition of Vectors

One method for adding vectors involves manipulating their graphical representations on paper. To do so, you need a ruler to measure and draw the vectors to the correct length, and a protractor to measure the angle that establishes the direction. The length of the arrow should be proportional to the magnitude of the quantity being represented, so you must decide on a scale for your drawing. For example, you might let 1 cm on paper represent 1 km. The important thing is to choose a scale that produces a diagram of reasonable size with a vector about 5–10 cm long.

One route from home to school shown in **Figure 4–2** involves traveling 2 km south and then 4 km west. **Figure 4–3** shows how these two vectors can be added to give the resultant displacement, ***R.*** First, vector ***A*** is drawn pointing directly south. Then, vector ***B*** is drawn with the tail of ***B*** at the tip of ***A*** and pointing directly west. Finally, the resultant is drawn from the tail of ***A*** to the tip of ***B.*** The order of the addition can be reversed. Prove to yourself that the resultant would be the same if you drew ***B*** first and placed the tail of ***A*** at the tip of ***B.***

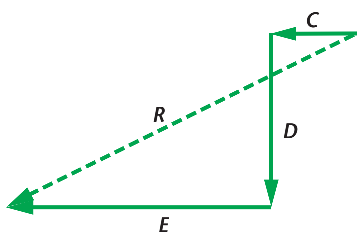
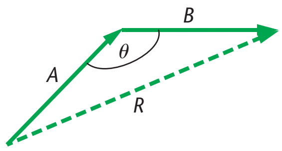


FIGURE 4–4 If you compare the displacement for route ***AB,*** shown in **Figure 4–3,** with the displacement for route ***CDE,*** you will find that the displacements are equal.

The magnitude of the resultant is found by measuring the length of the resultant with a ruler. To determine the direction, use a protractor to measure the number of degrees west of south the resultant is. How could you find the resultant vector of more than two vectors? **Figure 4–4** shows how to add the three vectors representing the second path you could take from home to school. Draw vector ***C,*** then place the tail of ***D***



$R^2 = A^2 + B^2 - 2AB\cos\theta$

FIGURE 4–5 The Law of Cosines is used to calculate the magnitude of the resultant when the angle between the vectors is other than 90°.

at the tip of ***C.*** The third vector, ***E,*** is added in the same way. Place the tail of ***E*** at the tip of ***D.*** The resultant, ***R,*** is drawn from the tail of ***C*** to the tip of ***E.*** Use the ruler to measure the magnitude and the protractor to find the direction. If you measure the lengths of the resultant vectors in **Figures 4–3** and **4–4,** you will find that even though the paths that were walked are different, the resulting displacements are equal.

The magnitude of the resultant If the two vectors to be added are at right angles, as shown in **Figure 4–3,** the magnitude can be found by using the Pythagorean theorem.

Pythagorean Theorem $R^2 = A^2 + B^2$

The magnitude of the resultant vector can be determined by calculating the square root. If the two vectors to be added are at some angle other than 90°, then you can use the Law of Cosines.

Law of Cosines $R^2 = A^2 + B^2 - 2AB\cos\theta$

This equation calculates the magnitude of the resultant vector from the known magnitudes of the vectors ***A*** and ***B*** and the cosine of the angle, θ, between them. **Figure 4–5** shows the vector addition of ***A*** and ***B.*** Notice that the vectors must be placed tail to tip, and the angle θ is the angle between them.

Math Handbook

To review the **Law of Cosines** and the **Law of Sines,** see the Math Handbook, Appendix A, page 746.

Example Problem

Finding the Magnitude of the Sum of Two Vectors

Find the magnitude of the sum of a 15-km displacement and a 25-km displacement when the angle between them is 135°.

Sketch the Problem

- **Figure 4–5** shows the two displacement vectors, ***A*** and ***B,*** and the angle between them.

Calculate Your Answer

Known:

$A = 25$ km

$B = 15$ km

$\theta = 135°$

Unknown:

$R = ?$

Strategy:

Use the Law of Cosines to find the magnitude of the resultant vector when the angle does not equal 90°.

Calculations:

$$\begin{aligned} R^2 &= A^2 + B^2 - 2AB\cos\theta \\ &= (25\text{ km})^2 + (15\text{ km})^2 - 2(25\text{ km})(15\text{ km})\cos 135° \\ &= 625\text{ km}^2 + 225\text{ km}^2 - 750\text{ km}^2(\cos 135°) \\ &= 1380\text{ km}^2 \\ R &= \sqrt{1380\text{ km}^2} \\ &= 37\text{ km} \end{aligned}$$

Check Your Answer

- Is the unit correct? The unit of the answer is a length.
- Does the sign make sense? The sum should be positive.
- Is the magnitude realistic? The magnitude is in the same range as the two combined vectors but longer than either of them, as it should be because the resultant is the side opposite an obtuse angle.

Practice Problems

1. A car is driven 125 km due west, then 65 km due south. What is the magnitude of its displacement?

2. A shopper walks from the door of the mall to her car 250 m down a lane of cars, then turns 90° to the right and walks an additional 60 m. What is the magnitude of the displacement of her car from the mall door?

3. A hiker walks 4.5 km in one direction, then makes a 45° turn to the right and walks another 6.4 km. What is the magnitude of her displacement?

4. What is the magnitude of your displacement when you follow directions that tell you to walk 225 m in one direction, make a 90° turn to the left and walk 350 m, then make a 30° turn to the right and walk 125 m?

Subtracting Vectors

Multiplying a vector by a scalar number changes its length but not its direction unless the scalar is negative. Then, the vector's direction is reversed. This fact can be used to subtract two vectors using the same methods you used for adding them. For example, you've learned that the difference in two velocities is defined by this equation.

$$\Delta \boldsymbol{v} = \boldsymbol{v}_2 - \boldsymbol{v}_1$$

The equation can be written as the sum of two vectors.

$$\Delta \boldsymbol{v} = \boldsymbol{v}_2 + (-\boldsymbol{v}_1)$$

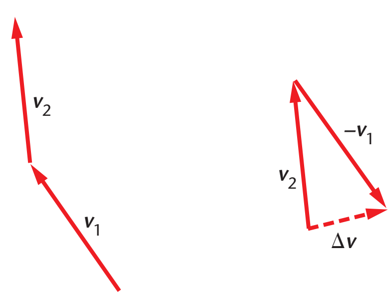


FIGURE 4–6 To subtract two vectors, reverse the direction of the second vector and then add them.

USING A CALCULATOR

Law of Cosines

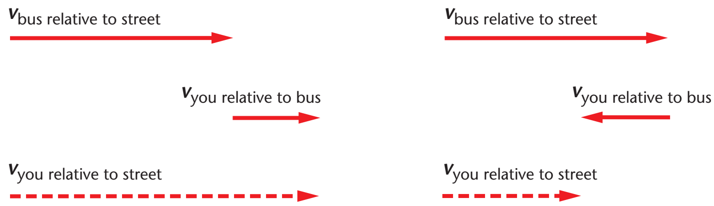
Use your calculator to solve for R using the Law of Cosines.

$R^2 = A^2 + B^2 - 2AB\cos\theta$
A = 25 km
B = 15 km
θ = 135°

Key	Result
√ (25 x²	625
+ 15 x²	850
− (2 × 25	
× 15 × cos 135	
)) =	37

Answer
37 km

FIGURE 4–7 When a coordinate system is moving, two velocities add if both motions are in the same direction and subtract if the motions are in opposite directions.



HELP WANTED

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If $\boldsymbol{v}_1$ is multiplied by -1, the direction of $\boldsymbol{v}_1$ is reversed as shown in **Figure 4–6.** The vector $-\boldsymbol{v}_1$ can then be added to $\boldsymbol{v}_2$ to get the resultant, which represents the difference, $\Delta\boldsymbol{v}$.

Relative Velocities: Some Applications

Graphical addition of vectors can be a useful tool when solving problems that involve relative velocity. Suppose you're in a school bus traveling at a velocity of 8 m/s in a positive direction. You walk at 3 m/s toward the front of the bus. How fast are you moving relative to the street? To solve this problem, you must translate these statements into symbols. If the bus is going 8 m/s, that means that the velocity of the bus is 8 m/s as measured in a coordinate system fixed to the street. Standing still, your velocity relative to the street is also 8 m/s but your velocity relative to the bus is zero. *Walking at 3 m/s toward the front of the bus* means that your velocity is measured relative to the bus. The question can be rephrased: Given the velocity of the bus relative to the street and your velocity relative to the bus, what is your velocity relative to the street?

A vector representation of this problem is shown in **Figure 4–7.** After looking at it and thinking about it, you'll agree that your velocity relative to the street is 11 m/s, the sum of 8 m/s and 3 m/s. Suppose you now walked at the same speed toward the rear of the bus. What would be your velocity relative to the street? **Figure 4–7** shows that because the two velocities are in opposite directions, the resultant velocity is 5 m/s, the difference between 8 m/s and 3 m/s. You can see that when the velocities are along the same line, simple addition or subtraction can be used to determine the relative velocity.

The addition of relative velocities can be extended to include motion in two dimensions. For example, airline pilots cannot expect to reach their destinations by simply aiming their planes along a compass direction. They must take into account the plane's velocity relative to the air, which is given by their airspeed indicators and their direction relative to the air. They must also consider the velocity of the wind that they must fly through relative to the ground. These two vectors must be combined, as shown in **Figure 4–8,** to obtain the velocity of the airplane relative to the ground. The resultant vector tells the pilot how fast and in what direction the plane must travel relative to the ground to reach its destination. You can add relative velocities even if they are at arbitrary angles by using a graphical method.

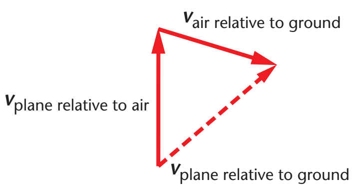


FIGURE 4–8 The plane's velocity relative to the ground can be obtained by vector addition.

CONTENTS

The Paper River

Problem

How does a boat travel on a river?

Materials

small battery-powered car (or physics bulldozer)
meterstick
protractor
stopwatch
a piece of paper, 1 m × 10 m

Procedure

1. Your car will serve as the boat. Write a brief statement to explain how the boat's speed can be determined.
2. Your boat will start with all wheels on the paper river. Measure the width of the river and predict how much time is needed for your boat to go directly across the river. Show your data and calculations.
3. Determine the time needed to cross the river when your boat is placed on the edge of the river. Make three trials and record the times.
4. Using the average of your trials, construct a graph showing the position and time for the boat crossing the river. If possible, use a computer or calculator to create the graph. Use this graph to observe and identify the relationship between variables.
5. Do you think it will take the boat more or less time to cross the river when the river is flowing? Explain your prediction.
6. Have a student (the hydro engineer) walk slowly at a constant speed, pulling the river along the floor. Each group should measure the time it takes for the boat to cross the flowing river. Make three trials and record the times. Compare the results with your prediction.
7. Using the grid from Step 4 and the average of your data from Step 6, construct a graph showing the position and time for the boat crossing the river when the river is flowing. Use a different color for the plot than you did for the boat without the river flowing.
8. Devise a method to measure the speed of the river. Have the hydro engineer pull the river at a constant speed and collect the necessary data.
9. Save the paper for later classes to use, or recycle it.

Data and Observations

1. Does the boat move in the direction that it is pointing?
2. Analyze and evaluate the trends in your data. How did the graphs of position versus time compare?
3. Infer from the trends in your data if the motion of the water affected the time needed to cross when the boat was pointed straight to the far shore.
4. Based on the trends in your data, predict whether the river or the boat had the greater speed. Explain your choice.

Analyze and Conclude

1. **Calculating Results** Calculate the speed of the river.
2. **Inferring Conclusions** Using your results for the speed of the boat and the speed of the river, calculate the speed of the boat compared to the ground when the boat is headed directly downstream and directly upstream.

Apply

1. Do small propeller aircraft always move in the direction that they are pointing? Do they ever fly sideways?
2. Try this lab again using a battery-powered boat on a small stream.

Physics & Society

Assessing Risk

Nearly every decision you make involves risk. Risk is the likelihood that a decision you make will cause you, another person, or an object injury, damage, or even loss. Read the information below and assess whether you think air bags should be standard equipment in automobiles.

Air Bags—Assets or Assaults?

Air bags are designed to be protective cushions between a front-seat occupant and the car's steering column or dashboard. About 50 percent of the cars and light trucks now on U.S. roads have driver's-side air bags. About 37 percent of these vehicles also have passenger-side air bags. By 1999, all new passenger cars and trucks sold in the United States were required to have passenger, as well as driver's-side, air bags.

From the late 1980s until late 1999, approximately 3.8 million air bags were deployed. The National Highway Traffic Safety Administration estimates that fatalities to car and light-truck drivers as well as car passengers have been cut by a third as a result of air bag deployment.

However, air bags have been responsible for the deaths of 165 people, including 97 children, who might have otherwise survived the crash. Because air bags inflate at speeds up to 200 km/h (124 mph), the energy associated with deployment can injure drivers and passengers who are too close to the air bag. These fatalities have prompted safety experts to recommend that children under the age of 12 never ride in the front seat.

Proponents of automobile air bags admit that there is a risk, but believe that the number of lives saved is sufficient reason for the installation of air bags in all vehicles. Suggested design changes include sensors to assess the severity of the impact and determine the weight and location of front-seat occupants at the time of the crash. With these data, a "smart" air bag could decrease the force with which the air bags deploy. A smart air bag might even prevent deployment if the driver or passenger was in danger of being injured by the air bag.

Air bag opponents contend that there is still no system that takes into account every possible crash scenario. Many opponents feel that the federal government moved too quickly when it legislated the installation of air bags. Opponents also argue that air bag regulations are biased because they require the air bag to protect an unbelted 77-kg (170-lb) male. Some opponents propose that air bags be optional equipment or that people should have the choice of disabling air bags.

Investigating the Issue

1. **Debating the Issue** Review, analyze, and critique the hypothesis that, overall, air bags save lives rather than cause deaths. Be sure to include the strengths and weaknesses of the hypothesis.
2. **Acquiring Information** Find out more about air bag research. Evaluate the impact of air bag research on society. Do you think the research is beneficial?
3. **Thinking Critically** Would today's air bags be useful in a rear-end collision? Explain.

PHYSICS Online

CLICK HERE

To find out more about air bags, visit the Glencoe Science Web site at science.glencoe.com

Practice Problems

5. A car moving east at 45 km/h turns and travels west at 30 km/h. What are the magnitude and direction of the change in velocity?
6. You are riding in a bus moving slowly through heavy traffic at 2.0 m/s. You hurry to the front of the bus at 4.0 m/s relative to the bus. What is your speed relative to the street?
7. A motorboat heads due east at 11 m/s relative to the water across a river that flows due north at 5.0 m/s. What is the velocity of the motorboat with respect to the shore?
8. A boat is rowed directly upriver at a speed of 2.5 m/s relative to the water. Viewers on the shore find that it is moving at only 0.5 m/s relative to the shore. What is the speed of the river? Is it moving with or against the boat?
9. An airplane flies due north at 150 km/h with respect to the air. There is a wind blowing at 75 km/h to the east relative to the ground. What is the plane's speed with respect to the ground?
10. An airplane flies due west at 185 km/h with respect to the air. There is a wind blowing at 85 km/h to the northeast relative to the ground. What is the plane's speed with respect to the ground?

F.Y.I.

Vector is a term used in biology and medicine to describe any disease-carrying microorganism. In genetics, a vector is any self-replicating DNA molecule that will carry one gene from one organism to another.

4.1 Section Review

1. Is the distance you walk equal to the magnitude of your displacement? Give an example that supports your conclusion.
2. A fishing boat with a maximum speed of 3 m/s with respect to the water is in a river that is flowing at 2 m/s. What is the maximum speed of the boat with respect to the shore? The minimum speed? Give the direction of the boat, relative to the river's current, for the maximum speed and the minimum speed relative to the shore.
3. The order in which vectors are added doesn't matter. Mathematicians say that vector addition is commutative. Which ordinary arithmetic operations are commutative? Which are not?
4. **Critical Thinking** A box is moved through one displacement and then through a second displacement. The magnitudes of the two displacements are unequal. Could the displacements have directions such that the resultant displacement is zero? Suppose the box was moved through three displacements of unequal magnitude? Could the resultant displacement be zero? Support your argument with a diagram.

4.2 Components of Vectors

The graphical method of adding vectors did not require that you decide on a coordinate system. The sum, or the difference, of vectors is the same no matter what coordinate system is used. Nevertheless, as you'll find, creating and using a coordinate system allows you not only to make quantitative measurements, but also provides an alternative method of adding vectors.

OBJECTIVES

- **Establish** a coordinate system in problems involving vector quantities.
- **Use** the process of resolution of vectors to find the components of vectors.
- **Determine** algebraically the sum of two or more vectors by adding the components of the vectors.

Choosing a Coordinate System

Choosing a coordinate system, such as the one in **Figure 4–9a,** is similar to laying a grid drawn on a sheet of transparent plastic on top of your problem. You have to choose where to put the center of the grid (the origin) and establish the direction in which the axes point. Notice that in the coordinate system shown in **Figure 4–9a,** the x-axis is drawn through the origin with an arrow pointing in the positive direction. Then, the positive y-axis is located 90° counterclockwise from the positive x-axis and crosses the x-axis at the origin.

How do you choose the direction of the x-axis? There is never a single correct answer, but some choices make the problem easier to solve than others. When the motion you are describing is confined to the surface of Earth, it is often convenient to have the x-axis point east and the y-axis point north. When the motion involves an object moving through the air, the positive x-axis is often chosen to be horizontal and the positive y-axis vertical (upward). If the motion is on a hill, it's convenient to place the positive x-axis in the direction of the motion and the y-axis perpendicular to the x-axis.

After the coordinate system is chosen, the direction of any vector can be specified relative to those coordinates. The direction of a vector is defined as the angle that the vector makes with the x-axis, measured counterclockwise. In **Figure 4–9b,** the angle θ tells the direction of the vector $\boldsymbol{A}$.

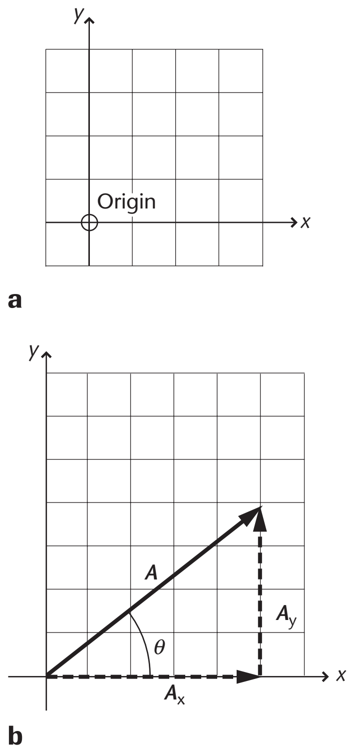


FIGURE 4–9 A coordinate system has an origin and two perpendicular axes, as in **a.** In **b,** the direction of a vector is measured counterclockwise from the x-axis.

Components

A coordinate system allows you to expand your description of a vector. In the coordinate system shown in **Figure 4–9b,** the vector $\boldsymbol{A}$ is broken up or resolved into two component vectors. One, $\boldsymbol{A}_x$, is parallel to the x-axis, and the other, $\boldsymbol{A}_y$, is parallel to the y-axis. You can see that the original vector is the sum of the two component vectors.

$$\boldsymbol{A} = \boldsymbol{A}_x + \boldsymbol{A}_y$$

The process of breaking a vector into its components is sometimes called **vector resolution.** The magnitude and sign of component vectors are called the **components.** All algebraic calculations involve only the components of vectors, not the vectors themselves. You can find the components by using trigonometry. The components are calculated according to these equations, where the angle θ is measured counterclockwise from the positive x-axis.

Component Vectors

$$A_x = A\cos\theta\text{; therefore, }\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{A_x}{A}$$

$$A_y = A\sin\theta\text{; therefore, }\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{A_y}{A}$$

When the angle that a vector makes with the x-axis is larger than 90°—that is, the vector is in the second, third, or fourth quadrants—the sign of one or more components is negative, as shown in **Figure 4–10.** Although the components are scalars, they can have both positive and negative signs.

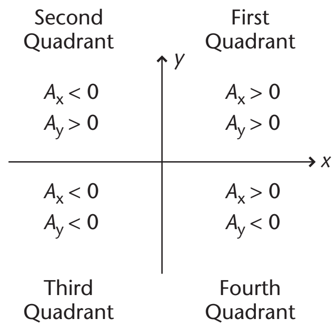
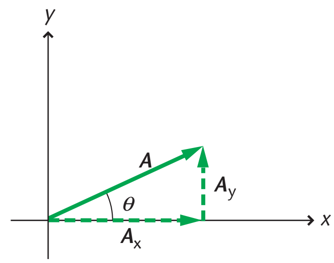


FIGURE 4–10 The sign of a component depends upon which of the four quadrants the component is in.

Example Problem

The Components of Displacement

A bus travels 23.0 km on a straight road that is 30° north of east. What are the east and north components of its displacement?



Sketch the Problem

- Draw the same sketch as in **Figure 4–9b.**
- A coordinate system is used in which the x-axis points east.
- The angle θ is measured counterclockwise from the x-axis.

Calculate Your Answer

Known:

$A = 23.0$ km

$\theta = 30°$

Unknown:

$A_x = ?$

$A_y = ?$

Strategy:

Use the trigonometric ratios to find the components.

Calculations:

$A_x = A\cos\theta$

$A_y = A\sin\theta$

$A_x = (23.0\text{ km})\cos 30°$
$= +19.9$ km

$A_y = (23.0\text{ km})\sin 30°$
$= +11.5$ km

Check Your Answer

- Are the units correct? The kilometer is an appropriate unit of length.
- Do the signs make sense? Both components are in the first quadrant and should be positive.
- Are the magnitudes reasonable? The magnitudes are less than the hypotenuse of the right triangle of which they are the other two sides.

Practice Problems

11. What are the components of a vector of magnitude 1.5 m at an angle of 35° from the positive *x*-axis?

12. A hiker walks 14.7 km at an angle 35° south of east. Find the east and north components of this walk.

13. An airplane flies at 65 m/s in the direction 149° counterclockwise from east. What are the east and north components of the plane's velocity?

14. A golf ball, hit from the tee, travels 325 m in a direction 25° south of the east axis. What are the east and north components of its displacement?

Pocket Lab

Ladybug

You notice a ladybug moving from one corner of your textbook to the corner diagonally opposite. The trip takes the ladybug 6.0 s. Use the long side of the book as the *x*-axis. Find the component vectors of the ladybug's velocity, $\boldsymbol{v}_x$ and $\boldsymbol{v}_y$, and the resultant velocity $\boldsymbol{R}$.

Analyze and Conclude Does the ladybug's path from one corner to the other affect the values in your measurements or calculations? Do $\boldsymbol{v}_x + \boldsymbol{v}_y$ really add up to $\boldsymbol{R}$? Explain.

Algebraic Addition of Vectors

Two or more vectors (***A, B, C,***. . .) may be added by first resolving each vector to its *x*- and *y*-components. The *x*-components are added to form the *x*-component of the resultant, $R_x = A_x + B_x + C_x + \ldots$ Similarly, the *y*-components are added to form the *y*-component of the resultant, $R_y = A_y + B_y + C_y + \ldots$.

The process is illustrated graphically in **Figure 4–11.** Because R_x and R_y are at a right angle (90°), the magnitude of the resultant vector can be calculated using the Pythagorean theorem.

$$R^2 = R_x^{\,2} + R_y^{\,2}$$

To find the angle or direction of the resultant, recall that the tangent of the angle that the vector makes with the *x*-axis is given by the following.

Angle of Resultant Vector $\tan\theta = \dfrac{R_y}{R_x}$

You can find the angle by using the $\tan^{-1}$ key on your calculator. Note: when $\tan\theta > 0$, most calculators give the angle between 0 and 90°; when $\tan\theta < 0$, the angle is reported to be between 0 and −90°.

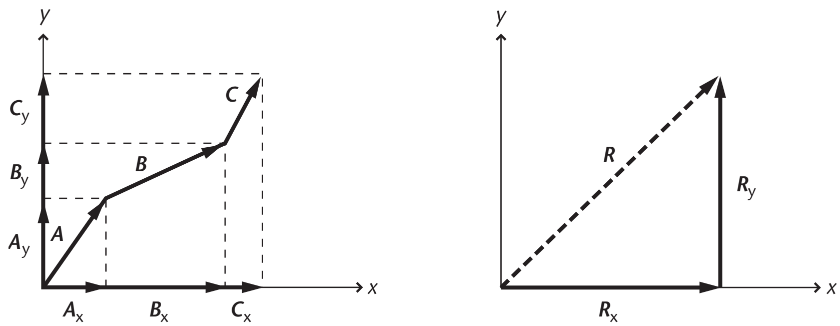


FIGURE 4–11 R_x is the sum of the *x*-components of ***A, B,*** and ***C.*** R_y is the sum of the *y*-components. The vector sum of R_x and R_y is the vector sum of ***A, B,*** and ***C.***

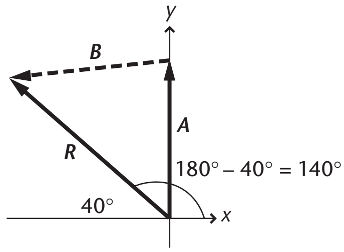
Example Problem

Finding Your Way Home

A GPS receiver told you that your home was 15.0 km at a direction of 40° north of west, but the only path led directly north. If you took that path and walked 10.0 km, how far, and in what direction would you then have to walk to reach your home?

Sketch the Problem

- Draw the resultant vector, ***R*** from your original location to home.
- Draw ***A***, the known displacement.
- Draw ***B***, the unknown displacement.



Calculate Your Answer

Known:

A = 10.0 km, due north

R = 15.0 km, 40° north of west

Unknown:

B = ?

Strategy:

Find the components of ***R*** and ***A***.

Calculations:

$R_x = R\cos\theta$
$= (15.0\text{ km})\cos 140°$
$= -11.5\text{ km}$

$R_y = R\sin\theta$
$= (15.0\text{ km})\sin 140°$
$= +9.6\text{ km}$

$A_x = 0.0\text{ km},\ A_y = 10.0\text{ km}$

Use the components of ***R*** and ***A*** to find the components of ***B***. The signs of B_x and B_y will tell you the direction of the component.

$\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B}$, so $\boldsymbol{B} = \boldsymbol{R} - \boldsymbol{A}$

$B_x = R_x - A_x = -11.5\text{ km} - 0.0\text{ km} = -11.5\text{ km}$;
This component points west.

$B_y = R_y - A_y = 9.6\text{ km} - 10.0\text{ km} = -0.4\text{ km}$;
This component points south.

Use the components of ***B*** to find the magnitude of ***B***.

$$B = \sqrt{B_x{}^2 + B_y}$$
$$= \sqrt{(-11.5\text{ km})^2 + (-0.4\text{ km})^2}$$
$$= 11.5\text{ km}$$

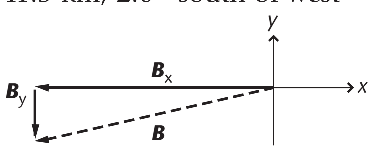
Use the tangent to find the direction of ***B***.

$$\tan\theta = \frac{B_y}{B_x} = \frac{-0.4\text{ km}}{-11.5\text{ km}} = +0.035$$

$$\theta = \tan^{-1}(+0.035) = 2.0°$$

Locate the tail of ***B*** at the origin of a coordinate system and draw the components $\boldsymbol{B}_x$ and $\boldsymbol{B}_y$. The direction is in the third quadrant, 2.0° south of west.

B = 11.5 km, 2.0° south of west



Continued on next page

Homeward Bound

Answers question from page 62.

CONTENTS

Check Your Answer

- Are the units correct? Kilometers and degrees are correct.
- Do the signs make sense? They agree with the diagram.
- Is the magnitude realistic? The length of ***B*** is reasonable because the angle between ***A*** and ***B*** is slightly less than 90°. If the angle were 90°, ***B*** would have been 11.2 km, which is close to 11.5 km. The direction of ***B*** deviates only slightly from the east-west direction.

Practice Problems

15. A powerboat heads due northwest at 13 m/s with respect to the water across a river that flows due north at 5.0 m/s. What is the velocity (both magnitude and direction) of the motorboat with respect to the shore?

16. An airplane flies due south at 175 km/h with respect to the air. There is a wind blowing at 85 km/h to the east relative to the ground. What are the plane's speed and direction with respect to the ground?

17. An airplane flies due north at 235 km/h with respect to the air. There is a wind blowing at 65 km/h to the northeast with respect to the ground. What are the plane's speed and direction with respect to the ground?

18. An airplane has a speed of 285 km/h with respect to the air. There is a wind blowing at 95 km/h at 30° north of east with respect to Earth. In which direction should the plane head in order to land at an airport due north of its present location? What would be the plane's speed with respect to the ground?

F.Y.I.

Although Oliver Heaviside was greatly respected by scientists of his day, he is almost forgotten today. His methods of describing forces by means of vectors were so successful that they were used in textbooks by other people. Unfortunately, few gave Heaviside credit for his work.

4.2 Section Review

1. You first walk 8.0 km north from home, then walk east until your distance from home is 10.0 km. How far east did you walk?
2. Could a vector ever be shorter than one of its components? Equal in length to one of its components? Explain.
3. In a coordinate system in which the *x*-axis is east, for what range of angles is the *x*-component positive? For what range is it negative?
4. **Critical Thinking** You are piloting a boat across a fast-moving river. You want to reach a pier directly opposite your starting point. Describe how you would select your heading in terms of the components of your velocity relative to the water.

CHAPTER 4 REVIEW

Key Terms

4.1

- graphical representation
- algebraic representation
- resultant vector

4.2

- vector resolution
- component

Summary

4.1 Properties of Vectors

- Vectors are quantities that have both magnitude and direction. They can be represented graphically as arrows or algebraically as symbols.
- Vectors are not changed by moving them, as long as their magnitudes (lengths) and directions are maintained.
- Vectors can be added graphically by placing the tail of one at the tip of the other and drawing the resultant from the tail of the first to the tip of the second.
- The sum of two or more vectors is the resultant vector.
- The Law of Cosines may be used to find the magnitude of the resultant of any two vectors. This simplifies to the Pythagorean theorem if the vectors are at right angles.
- Vector addition may be used to solve problems involving relative velocities.

4.2 Components of Vectors

- Placing vectors in a coordinate system that you have chosen makes it possible to decompose them into components along each of the chosen coordinate axes.
- The components of a vector are the projections of the component vectors. They are scalars and have signs, positive or negative, indicating their directions.
- Two or more vectors can be added by separately adding the *x*- and *y*-components. These components can then be used to determine the magnitude and direction of the resultant vector.

Key Equations

4.1

$R^2 = A^2 + B^2$

$R^2 = A^2 + B^2 - 2AB\cos\theta$

4.2

$A_x = A\cos\theta$; therefore, $\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{A_x}{A}$

$\tan\theta = \dfrac{R_y}{R_x}$

$A_y = A\sin\theta$; therefore, $\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{A_y}{A}$

Reviewing Concepts

Section 4.1

1. Describe how you would add two vectors graphically.
2. Which of the following actions is permissible when you are graphically adding one vector to another: move the vector, rotate the vector, change the vector's length?
3. In your own words, write a clear definition of the resultant of two or more vectors. Do not tell how to find it, but tell what it represents.
4. How is the resultant displacement affected when two displacement vectors are added in a different order?
5. Explain the method you would use to subtract two vectors graphically.
6. Explain the difference between these two symbols: *A* and **A**.

Section 4.2

7. Describe a coordinate system that would be suitable for dealing with a problem in which a ball is thrown up into the air.
8. If a coordinate system is set up such that the positive x-axis points in a direction 30° above the horizontal, what should be the angle between the x-axis and the y-axis? What should be the direction of the positive y-axis?
9. The Pythagorean theorem is usually written $c^2 = a^2 + b^2$. If this relationship is used in vector addition, what do a, b, and c represent?
10. Using a coordinate system, how is the angle or direction of a vector determined with respect to the axes of the coordinate system?

Applying Concepts

11. A vector drawn 15 mm long represents a velocity of 30 m/s. How long should you draw a vector to represent a velocity of 20 m/s?
12. A vector that is 1 cm long represents a displacement of 5 km. How many kilometers are represented by a 3-cm vector drawn to the same scale?
13. What is the largest possible displacement resulting from two displacements with magnitudes 3 m and 4 m? What is the smallest possible resultant? Draw sketches to demonstrate your answers.
14. How does the resultant displacement change as the angle between two vectors increases from 0° to 180°?
15. A and B are two sides of a right triangle. If $\tan \theta = A/B$,
    a. which side of the triangle is longer if $\tan \theta$ is greater than one?
    b. which side is longer if $\tan \theta$ is less than one?
    c. what does it mean if $\tan \theta$ is equal to one?
16. A car has a velocity of 50 km/h in a direction 60° north of east. A coordinate system with the positive x-axis pointing east and a positive y-axis pointing north is chosen. Which component of the velocity vector is larger, x or y?
17. Under what conditions can the Pythagorean theorem, rather than the Law of Cosines, be used to find the magnitude of a resultant vector?
18. A problem involves a car moving up a hill so a coordinate system is chosen with the positive x-axis parallel to the surface of the hill. The problem also involves a stone that is dropped onto the car. Sketch the problem and show the components of the velocity vector of the stone.

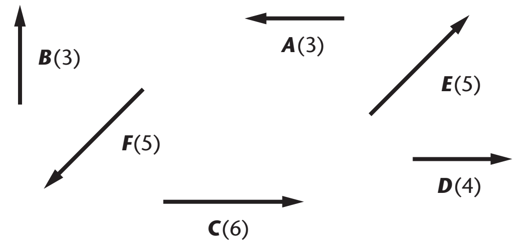


FIGURE 4–12

Problems

Section 4.1

19. A car moves 65 km due east, then 45 km due west. What is its total displacement?
20. Graphically find the sum of the following pairs of vectors whose lengths and directions are shown in **Figure 4–12.**
    a. ***D*** and ***A***
    b. ***C*** and ***D***
    c. ***C*** and ***A***
    d. ***E*** and ***F***
21. An airplane flies at 200.0 km/h with respect to the air. What is the velocity of the plane relative to the ground if it flies with
    a. a 50-km/h tailwind?
    b. a 50-km/h head wind?
22. Graphically add the following sets of vectors as shown in **Figure 4–12.**
    a. ***A, C,*** and ***D***
    b. ***A, B,*** and ***E***
    c. ***B, D,*** and ***F***
23. Path A is 8.0 km long heading 60.0° north of east. Path B is 7.0 km long in a direction due east. Path C is 4.0 km long heading 315° counterclockwise from east.
    a. Graphically add the hiker's displacements in the order ***A, B, C.***
    b. Graphically add the hiker's displacements in the order ***C, B, A.***
    c. What can you conclude about the resulting displacements?

24. A river flows toward the east. Because of your knowledge of physics, you head your boat 53° west of north and have a velocity of 6.0 m/s due north relative to the shore.
   a. What is the velocity of the current?
   b. What is your speed relative to the water?

Section 4.2

25. You walk 30 m south and 30 m east. Find the magnitude and direction of the resultant displacement both graphically and algebraically.
26. A ship leaves its home port expecting to travel to a port 500.0 km due south. Before it moves even 1 km, a severe storm blows it 100.0 km due east. How far is the ship from its destination? In what direction must it travel to reach its destination?
27. A descent vehicle landing on Mars has a vertical velocity toward the surface of Mars of 5.5 m/s. At the same time, it has a horizontal velocity of 3.5 m/s.
   a. At what speed does the vehicle move along its descent path?
   b. At what angle with the vertical is this path?
28. You are piloting a small plane, and you want to reach an airport 450 km due south in 3.0 hours. A wind is blowing from the west at 50.0 km/h. What heading and airspeed should you choose to reach your destination in time?
29. A hiker leaves camp and, using a compass, walks 4 km E, then 6 km S, 3 km E, 5 km N, 10 km W, 8 km N, and finally 3 km S. At the end of three days, the hiker is lost. By drawing a diagram, compute how far the hiker is from camp and which direction should be taken to get back to camp.
30. You row a boat perpendicular to the shore of a river that flows at 3.0 m/s. The velocity of your boat is 4.0 m/s relative to the water.
   a. What is the velocity of your boat relative to the shore?
   b. What is the component of your velocity parallel to the shore? Perpendicular to it?
31. A weather station releases a balloon that rises at a constant 15 m/s relative to the air, but there is a wind blowing at 6.5 m/s toward the west. What are the magnitude and direction of the velocity of the balloon?

Extra Practice For more practice solving problems, go to Extra Practice Problems, Appendix B.

Critical Thinking Problems

32. An airplane, moving at 375 m/s relative to the ground, fires a missile forward at a speed of 782 m/s relative to the plane. What is the speed of the missile relative to the ground?
33. A rocket in outer space that is moving at a speed of 1.25 km/s relative to an observer fires its motor. Hot gases are expelled out the rear at 2.75 km/s relative to the rocket. What is the speed of the gases relative to the observer?

Going Further

Albert Einstein showed that the rule you learned for the addition of velocities doesn't work for objects moving near the speed of light. For example, if a rocket moving at velocity v_A releases a missile that has a velocity v_B relative to the rocket, then the velocity of the missile relative to an observer that is at rest is given by,

$$v = \frac{v_A + v_B}{1 + v_A v_B / c^2}$$ where c is the speed of light,

3.00×10^8 m/s. This formula gives the correct values for objects moving at slow speeds as well. Suppose a rocket moving at 11 km/s shoots a laser beam out front. What speed would an unmoving observer find for the laser light? Suppose a rocket moves at a speed of $c/2$, half the speed of light, and shoots a missile forward at a speed of $c/2$ relative to the rocket. How fast would the missile be moving relative to a fixed observer?

PHYSICS Online

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